One-Way ANOVA

SPSS for Windows® Intermediate & Advanced Applied Statistics

Zayed University Office of Research SPSS for Windows® Workshop Series
Presented by

Dr. Maher Khelifa
Associate Professor
Department of Humanities and Social Sciences
College of Arts and Sciences

Understanding One-Way ANOVA

- A common statistical technique for determining if differences exist between two or more "groups" is One-way Analysis of Variance.

- One-Way ANOVA tests whether the means of two or more independent groups are equal by analyzing comparisons of variance estimates.

Understanding One-Way ANOVA

- In general, however, the One-Way ANOVA is used to test for differences among three groups as comparing the means of two groups can be examined using an independent t-test.

- When there are only two means to compare, the t-test and the F-test are equivalent and generate the same results.

- This is why One-Way ANOVA is considered an extension of the independent t-test.

- This analysis can only be performed on numerical data (data with quantitative value).

Understanding One-Way ANOVA

- The One-Way ANOVA statistic tests the null hypothesis that samples in two or more groups are drawn from the same population.
  - The null hypothesis (H_o) will be that all sample means are equal (H_o: $\mu_1 = \mu_2 = \mu_3$).
  - the alternative hypothesis (H_A) is that at least one mean is different (Not H_o).
  - Decision rules: If F_{obs} is greater or equal to F_{crit}, reject H_o, otherwise do not reject $H_{o.}$
  - If the decision is to reject the null, then at least one of the means is different. However, the omnibus one-way ANOVA does not tell you where the difference lies. For this, you need post-hoc tests.

F Table

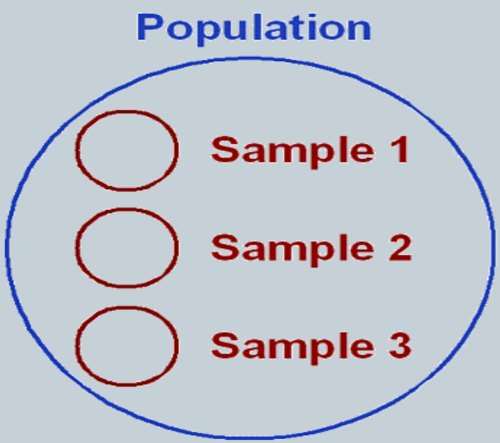
Critical values for alpha equals .05.

df_w \ df_B	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	245.9	248.0	249.1	250.1	251.1	252.2	253.3	254.3
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

One-Way ANOVA Logic

- The test is called ANOVA rather than multi-group means analysis because it compares group means using comparative analysis of variance estimates.

- The H_o assumes the group means are equal.



One-Way ANOVA Logic

- When we select three samples. Why would these sample means differ?

- There are **two logical reasons**:
  - Group Membership (i.e., the *treatment effect*).
  - Differences not due to group membership (i.e., chance or *sampling error*).

- The ANOVA is based on the fact that two independent estimates of the population variance can be obtained from the sample data.

  - $\frac{\text{treatment effect \& error}}{\text{error}}$ or $\frac{\text{between groups estimate}}{\text{within groups estimate}}$

One-Way ANOVA Logic

- Given the null hypothesis in this case (H_0: $\mu_1 = \mu_2 = \mu_3$), the two variance estimates should be equal.

- That is, since the null assumes no treatment effect, both variance estimates reflect error and their ratio will equal 1.

- To the extent that this ratio is larger than 1, it suggests a *treatment effect* (i.e., differences between the groups) and the H_0 should be rejected.

One-Way ANOVA Logic

- **The F Test** is simply the ratio of the two variance estimates (dividing the between group variance by the within group variance):

$$F = \frac{MS_{Between}}{MS_{Within}}$$

- (Where **MS** means Mean Square, **Between** means between group variation, and **Within** means within group variation).

One-Way ANOVA

- In One-Way ANOVA each case must have scores on two variables: one single factor and one quantitative dependent variable.
- The factor divides individuals in two 2 or more groups or levels.
- The dependent variable differentiates individuals on some quantitative dimension.
- The ANOVA *F* test evaluates whether the group means on the dependent variable differ significantly from each other.
- The test hypothesis is that the group means are equal.
- In addition to determining that differences exist among the means, you may want to know which means differ.

Follow-up Tests

- If the overall ANOVA is significant and a factor has more than two levels, follow up tests are usually conducted. The overall ANOVA is called **omnibus** test.
- Frequently, follow-up tests involve comparisons between pairs of group means (referred to as **contrasts or pairwise comparisons**).
- Example, if a factor has three levels, 3 pairwise comparisons might be conducted to compare the means of group 1 and 2, the means of group 1 and 3, and the means of group 2 and 3.
- Follow-up tests are called Post-hoc multiple comparisons.

Post Hoc Equal variance Assumed

- Bonferroni: uses t tests to perform pairwise comparisons between group means, but controls overall rate by setting the error rate for each test to the experimentwise error rate divided by the total number of tests. Hence the observed significance level is adjusted for the fact that multiple comparisons are being made.

- Tukey Test: Uses studentized range statistic to make all of the pairwise comparisons between groups. Sets the error rate at the experimentwise error rate for the collection of all the pairwise comparisons.

- Sheffe: Performs simultaneous joint pairwise comparisons for all possible pairwise combinations. Uses the F sampling distribution.

Equal Variance Not Assumed

- Dunnett's C: Pairwise comparison test based on the studendentized range. This test is appropriate when the variances are not equal.
- Tamhane's T2: Conservative pairwise comparisons test based on a t test. This test is appropriate when the variances are not equal.
- Games-Howell: Pairwise comparison test that is sometimes liberal. This test is also appropriate when the variances are not equal.

One-Way ANOVA Assumptions

- The results of a one-way ANOVA can be considered reliable as long as the following assumptions are met:
- **1. Assumption of Normality:** The dependent variable is normally distributed for each of the populations (or approximately normally distributed). The different populations are defined by each level of a factor.
- **2. Homogeneity of Variance Assumption:** Variances of populations are equal. The variances of the dependent variable are the same for all population.
- **3. Assumption of Independence:** Samples are assumed independent. The cases represent random samples from the populations and the scores on the test variable (dependent variable) are independent of each other.

Assumption Violation and Test Robustness

- **1. Normality Assumption: Robust**

    - The one-way ANOVA is relatively robust to violations of the normality Assumption. With fairly small, moderate, and large sample sizes, the test may yield reasonably accurate *p* values even when the normality assumption is violated.

    - Large sample sizes may be required to produce relatively valid *p* values if the population distributions are substantially not normal.

    - The power of one-way ANOVA test may be reduced considerably if the population distributions are specifically thick tailed or heavily skewed.

Assumption Violation and Test Robustness

- **2. Homogeneity of variance assumption: Non-Robust**

  - to the extent that this assumption is violated and the sample sizes differ among groups, the resulting *p* value for the omnibus test is not trustworthy.

  - Under these conditions, it is preferable to use statistics that do not assume equality of population variances such as the **Browne-Forsythe** or the **Welch statistic** (they are accessible in SPSS by selecting Analyze, compare means, One-way ANOVA, and options).

  - For post-hoc tests, the validity of the results is questionable if the population variances differ regardless of whether the sample sizes are equal or unequal. It is recommended to choose **Dunnett's C** procedure in instances where the variances are unequal.

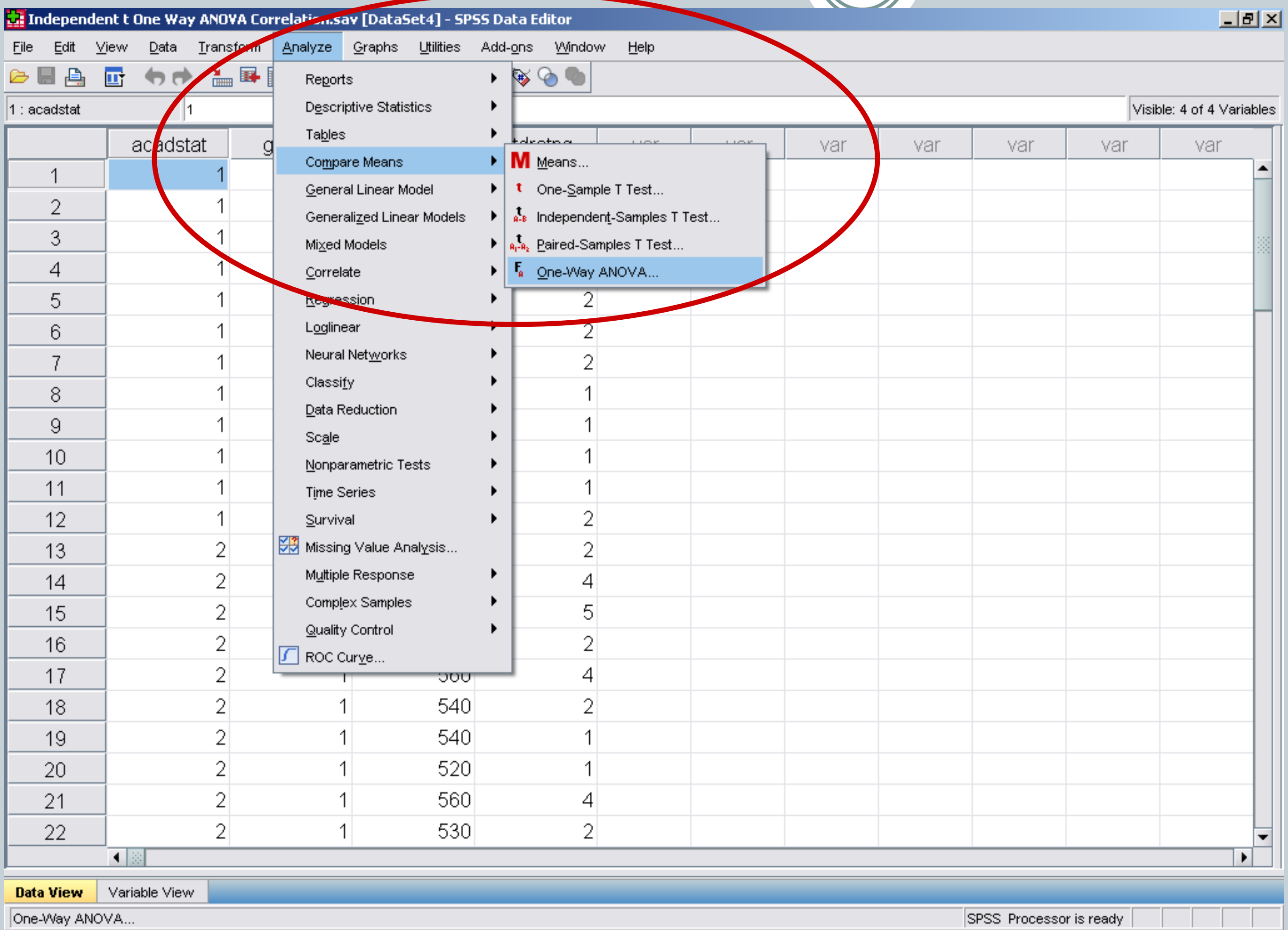
Assumption Violation and Test Robustness

- **3. Assumption of Independence: Non-Robust**
  - The ANOVA is not robust to the violation of independence assumption yielding inaccurate *p* values.

Alternate Tests

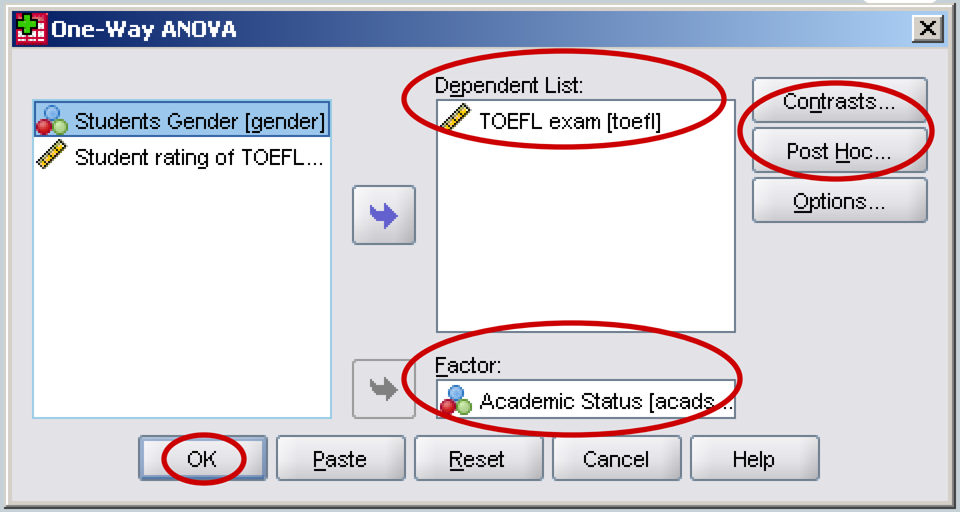
- If some of the assumption which if not met produce inaccurate p values or data are ordinal or non-parametric, alternative to this test should be used including Kruskal-Wallis one-way analysis of variance.

How to Obtain a One-Way ANOVA



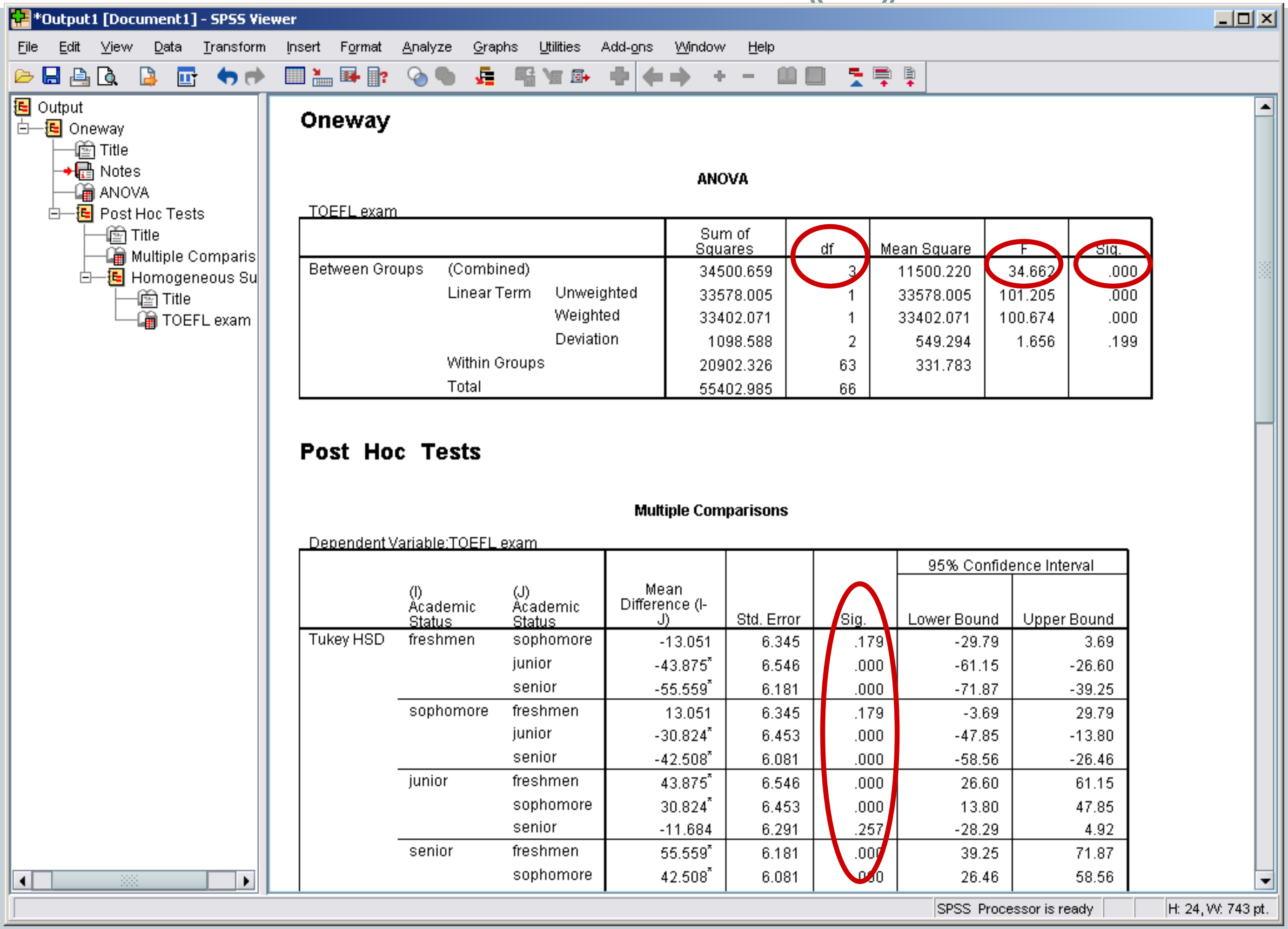
- Go to Analyze
- Compare Means
- One-Way Anova

How to Obtain a One-Way ANOVA



- Move test variable to dependent list box and the factor (grouping variable) to the factor box.
- If factor has more than 3 levels, press contrasts, and chose polynomial, then press Post hoc and choose Tukey, Sheffe, or Bonferroni.
- Then press OK

How to Obtain a One-Way ANOVA



Oneway

ANOVA

TOEFL exam

			Sum of Squares	df	Mean Square	F	Sig.
Between Groups	(Combined)		34500.659	3	11500.220	34.662	.000
	Linear Term	Unweighted	33578.005	1	33578.005	101.205	.000
		Weighted	33402.071	1	33402.071	100.674	.000
		Deviation	1098.588	2	549.294	1.656	.199
	Within Groups		20902.326	63	331.783		
	Total		55402.985	66			

Post Hoc Tests

Multiple Comparisons

Dependent Variable:TOEFL exam

	(I) Academic Status	(J) Academic Status	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Tukey HSD	freshmen	sophomore	-13.051	6.345	.179	-29.79	3.69
		junior	-43.875*	6.546	.000	-61.15	-26.60
		senior	-55.559*	6.181	.000	-71.87	-39.25
	sophomore	freshmen	13.051	6.345	.179	-3.69	29.79
		junior	-30.824*	6.453	.000	-47.85	-13.80
		senior	-42.508*	6.081	.000	-58.56	-26.46
	junior	freshmen	43.875*	6.546	.000	26.60	61.15
		sophomore	30.824*	6.453	.000	13.80	47.85
		senior	-11.684	6.291	.257	-28.29	4.92
	senior	freshmen	55.559*	6.181	.000	39.25	71.87
		sophomore	42.508*	6.081	.000	26.46	58.56

- In the output look for the F value, the df, and sig.
- In the Post hoc look for the level of significance of each pairwise comparison.

Bibliographical References

- Almar, E.C. (2000). Statistical Tricks and traps. Los Angeles, CA: Pyrczak Publishing.
- Bluman, A.G. (2008). Elemtary Statistics (6^{th} Ed.). New York, NY: McGraw Hill.
- Chatterjee, S., Hadi, A., & Price, B. (2000) *Regression analysis by example*. New York: Wiley.
- Cohen, J., & Cohen, P. (1983). Applied multiple regression/correlation analysis for the behavioral sciences (2^{nd} Ed.). Hillsdale, NJ.: Lawrence Erlbaum.
- Darlington, R.B. (1990). Regression and linear models. New York: McGraw-Hill.
- Einspruch, E.L. (2005). An introductory Guide to SPSS for Windows (2^{nd} Ed.). Thousand Oak, CA: Sage Publications.
- Fox, J. (1997) *Applied regression analysis, linear models, and related methods*. Thousand Oaks, CA: Sage Publications.
- Glassnapp, D. R. (1984). Change scores and regression suppressor conditions. Educational and Psychological Measurement (44), 851-867.
- Glassnapp. D. R., & Poggio, J. (1985). Essentials of Statistical Analysis for the Behavioral Sciences. Columbus, OH: Charles E. Merril Publishing.
- Grimm, L.G., & Yarnold, P.R. (2000). Reading and understanding Multivariate statistics. Washington DC: American Psychological Association.
- Hamilton, L.C. (1992) *Regression with graphics*. Belmont, CA: Wadsworth.
- Hochberg, Y., & Tamhane, A.C. (1987). Multiple Comparisons Procedures. New York: John Wiley.
- Jaeger, R. M. Statistics: A spectator sport (2^{nd} Ed.). Newbury Park, London: Sage Publications.

Bibliographical References

- Keppel, G. (1991). Design and Analysis: A researcher's handbook (3rd Ed.). Englwood Cliffs, NJ: Prentice Hall.
- Maracuilo, L.A., & Serlin, R.C. (1988). Statistical methods for the social and behavioral sciences. New York: Freeman and Company.
- Maxwell, S.E., & Delaney, H.D. (2000). Designing experiments and analyzing data: Amodel comparison perspective. Mahwah, NJ. : Lawrence Erlbaum.
- Norusis, J. M. (1993). SPSS for Windows Base System User's Guide. Release 6.0. Chicago, IL: SPSS Inc.
- Norusis, J. M. (1993). SPSS for Windows Advanced Statistics. Release 6.0. Chicago, IL: SPSS Inc.
- Norusis, J. M. (1994). SPSS Professional Statistics 6.1 . Chicago, IL: SPSS Inc.
- Norusis, J. M. (2006). *SPSS Statistics 15.0 Guide to Data Analysis*. Upper Saddle River, NJ.: Prentice Hall.
- Norusis, J. M. (2008). *SPSS Statistics 17.0 Guide to Data Analysis*. Upper Saddle River, NJ.: Prentice Hall.
- Norusis, J. M. (2008). *SPSS Statistics 17.0 Statistical Procedures Companion*. Upper Saddle River, NJ.: Prentice Hall.
- Norusis, J. M. (2008). *SPSS Statistics 17.0 Advanced Statistical Procedures Companion*. Upper Saddle River, NJ.: Prentice Hall.
- Pedhazur, E.J. (1997). *Multiple regression in behavioral research, third edition*. New York: Harcourt Brace College Publishers.

Bibliographical References

- SPSS Base 7.0 Application Guide (1996). Chicago, IL: SPSS Inc.
- SPSS Base 7.5 For Windows User's Guide (1996). Chicago, IL: SPSS Inc.
- SPSS Base 8.0 Application Guide (1998). Chicago, IL: SPSS Inc.
- SPSS Base 8.0 Syntax Reference Guide (1998). Chicago, IL: SPSS Inc.
- SPSS Base 9.0 User's Guide (1999). Chicago, IL: SPSS Inc.
- SPSS Base 10.0 Application Guide (1999). Chicago, IL: SPSS Inc.
- SPSS Base 10.0 Application Guide (1999). Chicago, IL: SPSS Inc.
- SPSS Interactive graphics (1999). Chicago, IL: SPSS Inc.
- SPSS Regression Models 11.0 (2001). Chicago, IL: SPSS Inc.
- SPSS Advanced Models 11.5 (2002) Chicago, IL: SPSS Inc.
- SPSS Base 11.5 User's Guide (2002). Chicago, IL: SPSS Inc.
- SPSS Base 12.0 User's Guide (2003). Chicago, IL: SPSS Inc.
- SPSS 13.0 Base User's Guide (2004). Chicago, IL: SPSS Inc.
- SPSS Base 14.0 User's Guide (2005). Chicago, IL: SPSS Inc..
- SPSS Base 15.0 User's Guide (2007). Chicago, IL: SPSS Inc.
- SPSS Base 16.0 User's Guide (2007). Chicago, IL: SPSS Inc.
- SPSS Statistics Base 17.0 User's Guide (2007). Chicago, IL: SPSS Inc.