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The Simplest Dynamic General-Equilibrium Model of an Open Economy - A Comment

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Abstract

The model developed by Devarajan and Go (1998) presents the simplest possible general-equilibrium model of an open economy in which producers' and consumers' decisions are both intra- and intertemporally consistent. Unfortunately, there is possible leakage in that imported capital goods are taxes twice, yet these taxes do not show up fully in the government's budget constraint. Additionally, one of the proposed terminal conditions is implied by the other equations because of Walras' Law. Therefore, the model description is lacking an appropriate terminal condition. In this paper an alternative set of equations is presented that removes the possible leakage and has an additional terminal condition with respect to one of the stock variables. Consequently, the model is square in that the number of unknowns is equal to the number of equations. The simulation results of the adjusted model and the original model are compared. Although the differences in simulation results turn out to be minor for the 1990 data set of the Philippines as used in Devarajan and Go (1998), it is shown that leakage is likely to occur.

Keywords: Dynamic CGE, Open economy.

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1. Introduction

A one-sector dynamic computable general equilibrium (CGE) model is developed by Devarajan and Go (1998; henceforth DG) for an open economy. Various intertemporal consistent dynamic CGE models (like the DG model) have appeared in the literature. An example of such a model looking at macroeconomic policies is by Dogruel, Dogruel and Yeldan (2003). Models aimed at analyzing international trade issues are by Diao and Somwaru (2000), Kouparitsas (2001) and Ghosha and Rao (2002). Models extended with environmental issues are analyzed by Bye (2000), Kemfert and Welsch (2000), Dissou, Wendner (2001), Mac Leod and Souissi (2002), Böhringer (2004), and finally, Dellink and van Ierland (2006). The compelling characteristics of the DG model are the limited number of equations, yet it describes in a stylized manner the workings of an open economy. The data requirements to calibrate the model are rather modest. Only national accounts data is needed. Therefore, the model has considerable merit in terms of parsimony and ease of use, particularly for the non-technical policy analyst. However, close inspection of the model equations reveals two problems.

First, the imported capital good is taxed twice and one of these tax receipts is not accounted for in the budget constraint of the government. There is an import tariff for the imported capital good, and the good is taxed in the same way as the other spending components, like private consumption, government consumption and investment. This latter tax is not accounted for if one looks at the equation where all tax receipts of the government are listed. In the following section the validity of the overall accounting scheme of the model is restored by modifying the model equations slightly. By doing that the flow of money and goods in the economy are fully accounted for in the model. Additionally, the levels of the stocks present in the model, will over time, also be fully and correctly accounted for if there is no leakage present.

The second problem is the specification of the terminal condition. Because of Walras' Law, the proposed terminal condition of DG is implied by the other accounting relationships. A terminal conditions is added to DG such that both stock variables present in the model will have certain prescribed values in the long-term when the steady state is reached. Finally, the original model, together with the proposed modifications, will provide accurate new results that, without the modifications, would be inaccurate, although the extent of the inaccuracy is not known beforehand.

The two problems may seem minor. Yet, if one models the economy as a whole there should not be implicit leakages that are hard to recognize and therefore go unnoticed. Any unaccounted for leakage weakens the model results. In what follows, solutions to these problems are presented which restore the model to its integrity. The first issue of the double taxation is discussed in Section 2. The issue of the specification of the terminal condition is discussed in Section 3. Both issues are resolved and an alternative set of model equations is presented in the Appendix where all taxes appear as receipts to the government, and where the stocks present in the model have certain prescribed values in the long-term that make economic sense. The adjusted model results are then compared to the results of the original model formulation as in DG. For this, economic data for the year 1990 of the Philippines is used and this is the same data set as in DG. Section 4 contains the details of this comparison. Section 5 concludes the paper with a summary of the findings.

2. Problem 1: Double Taxation

In order to identify the extent of the problem, extensive use must be made of the equations contained in the Appendix to DG. Only those most needed are reintroduced in this Section. The Appendix to this paper contains firstly the proposed set of model equations together with lists of parameters and variables. Secondly, it contains the original equations of DG that need to be altered. Specifically, they are Equations A.20, and A.34 of the Appendix of DG. These two equations contain expressions for the price of the capital (PK_t) and the total level of tax collected (TAX_t). Equation A.20 reads (a_k is the coefficient of capital imports, PMK_t is the domestic price of capital imports, P_t is the price of supply and tx_t is the domestic indirect tax rate):

$$PK_t = [a_k \cdot PMK_t + (1 - a_k) \cdot P_t] \cdot (1 + tx_t) \qquad \text{(A.20)}$$

Equation A.34 contains, among other things, the total receipts of domestic indirect tax collected as (C_t is aggregate consumption, G_t is government consumption and J_t is total investment expenditures, including adjustment cost):

$$tx_t \cdot [P_t \cdot (C_t + G_t + J_t)]$$

If one multiplies the right and left hand side of the Equation A.20 with the level of total investment expenditures including adjustment cost (J_t), the result looks like this:

$$PK_t \cdot J_t = a_k \cdot J_t \cdot PMK_t + (1 - a_k) \cdot J_t \cdot P_t + a_k \cdot J_t \cdot PMK_t \cdot tx_t + tx_t \cdot P_t \cdot (1 - a_k) \cdot J_t$$

This simplifies to:

$$PK_t \cdot J_t = MK_t \cdot PMK_t + (1 + tx_t) \cdot P_t \cdot (1 - a_k) \cdot J_t + a_k \cdot J_t \cdot PMK_t \cdot tx_t$$

Using $MK_t = a_k \cdot J_t$ (as specified by DG in Equation A.39) where MK_t denotes capital imports. From Equation A.34 of DG it is clear that the total of indirect taxes on investment goods should be equal to $tx_t \cdot P_t \cdot J_t$. This can only be true if the prices PMK_t and P_t are equal to each other. There is a possible and likely inconsistency in the equations of DG as this is not likely to be the case.

Even if the prices PMK_t and P_t are equal, the imported capital goods are taxed twice as the price PMK_t is the price including the import tariff on imported capital goods (see

Equation A.18 of DG which reads $PMK_t = pmk_t^* \cdot (1 + tmk_t) \cdot er$). A way out is to change the equation A.20 to the following expression:

$$PK_t = a_k \cdot PMK_t + (1 - a_k) \cdot P_t \cdot (1 + tx_t)$$

Then, Equation A.34 has to be amended to incorporate $tx_t \cdot P_t \cdot (1 - a_k) \cdot J_t$ as one of the taxes collected by the government.

3. Problem 2: Terminal Conditions

One of the proposed terminal conditions of DG in the article is the following equation (Equation A.10 of the Appendix to DG, where tf denotes the final time period, Y_{tf} is net income spent on consumption, PC_{tf} is the price of domestic supply including indirect tax):

$$Y_{tf} = PC_{tf} \cdot C_{tf} \qquad \text{(A.10)}$$

If one examines the accounting relationships, one will notice that this terminal condition of DG is implied by these equations. This is the manifestation of Walras' Law. Look at the Equations 2, 3, 13, 14, 15, 18, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 36, and 39 of the Appendix. These equations together imply Equation 4. This Equation 4 is valid for all time periods, including the final time period considered. For the final time period, Equation 4 becomes Equation A.10, which is the terminal condition of DG. In effect the model description of DG is lacking one terminal condition.

There is also GAMS code (Brooke et al. 1998) available from DG where they implement the model and calibrate it with data of the Philippines for the year 1990. There they have an additional terminal condition stating that in the final time period foreign borrowing, or capital inflow (B_t) is equal to its original level. This in turn leads to the conclusion that in the long-term the stock of net foreign debt ($DEBT_t$) returns to its original base run value. DG are correct in specifying a terminal condition with respect to one of the stock variables present in the model. Actually, any terminal condition has to be specified for each stock variable in the model. See Mercenier and Michel (1994a, 1994b). By insisting that the stock variables have certain prescribed values or have to meet certain conditions one can have for instance growth in the capital stock. Otherwise, without that particular terminal condition, the capital stock would be depleted in the final time period considered in the model. Because of the stability of the model, either assumed or, because of the functional forms chosen (Chiang 1997), are the long-term characteristics of the model as expected. The capital stock and the stock of net foreign debt are constant at their respective steady state values.

An alternative terminal condition would be the steady state version of the equation describing the evolution over time of the net foreign debt. Look at Equation 16 of the

Appendix. The level of foreign borrowing, or capital inflow, will be equal to the so-called trend adjustment to debt term (d_{adj} or in a way the depreciation term of net foreign debt) multiplied by the level of net foreign debt:

$$B_{tf} = d_{adj} \cdot DEBT_{tf}$$

This equation then provides the economic subject to choose a certain level of net foreign debt. They can now weigh the benefits of a high level of net foreign debt (borrow heavily from Rest-of-the-World (ROW) to finance consumption) with the cost in terms of high debt service payments. Or they can choose to have a low level of net foreign debt and have benefits in terms of low debt service payments. As such this trend adjustment to debt is either a source of wealth (manna from heaven), if a country is indebted to the ROW, or a country's wealth vaporizes if it has large foreign investments. This is the price one has to pay as a modeller to have such a term present in the model purely because of mathematical reasons. After all, with this term one can have a steady state with a constant level of net foreign debt and a value for (B_t) foreign borrowing, or capital inflow, which is not equal to zero.

4. Differences between the Simulations

In the previous two Sections the equations of DG that need to be altered have been discussed. The original equations and the altered equations are listed in the Appendix. One equation can be left out as it is implied by the other equations because of Walras' Law. One should also remember that the stock variables, capital and net foreign debt (K_t and $DEBT_t$) have given values at the first time period. As the model is homogenous of degree one in prices, one of the prices can be fixed as the Price Numéraire. The resulting system of equations is now a square system of equations where the number of endogenous variables is equal to the number of equations.

To compare the outcomes of the original model of DG with leakage and the adjusted model, simulations of both models are compared. The differences in the simulations are due to the adjustments applied to certain equations of the model.

Differences in Parameter and Exogenous Variables Values

The tax rate tx_t is different as the tax base is changed. Initially, the formula for calculating the tx -tax rate reads:

$$tx_t = \frac{TX}{C + G + J - TX}$$

The tax rate is the level of tax collected divided by the tax base which is the difference between the items including the tax minus the tax itself. The new formula for tx_t is now:

$$tx_t = \frac{TX}{C + G + J \cdot (1 - a_k) - TX}$$

As the tax base is smaller (the denominator is smaller in value), the tax rate is higher. This leads to a higher PC_t price as it is defined as:

$$PC_t = P_t \cdot (1 + tx_t)$$

The real per capita level of total investment expenditures, including adjustment cost (J_t) will now be lower with this higher value for PC_t as it is a calculated as:

$$\frac{J_t}{PC_t \cdot RSF_t}$$

Here RSF_t is the size of the population. The same is true for real per capita private consumption and government consumption. In the sequel a different terminal condition for the stock of net foreign debt will be used. Instead of $B_{tf} = B_0$ the terminal condition is now $B_{tf} = d_{adj} \cdot DEBT_{tf}$. The level of net foreign debt and the level of foreign borrowing will not return necessarily to their original values as it does in DG.

To be able to compare results the adjusted model has to be re-calibrated and this is done by adjusting as little of the parameters of the model as possible while maintaining that the base run replicates the values for the Philippines in 1990. The parameters that have different values are listed in Table 1.

Table 1: Parameters that have a different value for the two models.

Parameter	Original model value	Adjusted model value
tx_t	0.0535	0.0587
α_v	2.1197	2.1387
δ_v	0.2662	0.2665
β	0.5000	0.4253
δ	0.0825	0.0835
a_k	0.3861	0.3786

The indirect tax rate has changed as discussed previously. The parameters of the CES function for value-added have changed, like the shift parameter (α_v) and the share parameter (δ_v).Two parameters in the adjustment cost function for investment also have to change (β and δ). As the level of total investment expenditures, including adjustment cost (J_t) has changed there will be a different value for investment I_t and consequently, a different value for the parameter a_k. For the exogenous variables, Table 2 shows how their value has changed while the adjusted model is re-calibrated.

Table 2: Exogenous variables that have a different value for the two models.

Exogenous Variable	Original model value	Adjusted model value
G_t	1.5560	1.5485
pmk_t^*	0.0360	0.0362

The level of government consumption (G_t) has changed as the price of domestic supply including indirect tax (PC_t) has changed. The second exogenous variable that has changed is the world price of capital imports (pmk_t^*) to allow for a new steady state. As some of the parameter values and some of the values for the exogenous variables have changed in the adjusted model it is expected that the endogenous variables change in the base run and there will be different results for any of the simulations carried out with the adjusted model. Additionally, the adjusted model has a different terminal condition and no leakage. All effects combined will most probably lead to a different outcome for the adjusted model compared to the results of the DG model. First, it is shown how the endogenous variables change for the base run and after that for the experiment with a higher export price.

Differences in Endogenous Variables Values for the Base Run Simulation

The following table (Table 3) shows for some of the major endogenous variables the base run value of the DG model and the adjusted model. The final column contains the percentage difference between the DG model outcome compared and the adjusted model outcome.

Table 3: Base run outcome for the original and adjusted model, together with the percentage change for the major endogenous variables.

	DG outcome	Adjusted model outcome	%
C_t	11.890	11.834	-0.467
J_t	3.940	3.999	1.508
E_t	4.820	4.815	-0.103
MK_t	1.520	1.514	-0.392
MN_t	4.140	4.141	0.020
$DEBT_t$	0.470	0.468	-0.369
K_t	44.170	43.631	-1.221
TAX_t	2.270	2.267	-0.131

The base run values are steady state values as they are constant per capita real values. The reported values for the endogenous variables are valid over the entire time horizon that is considered (the years 1990 up to 2059, just as in DG). While calibrating the DG model the prices PMK_t and P_t are equal and so there is no leakage in the base run solution of the DG model.

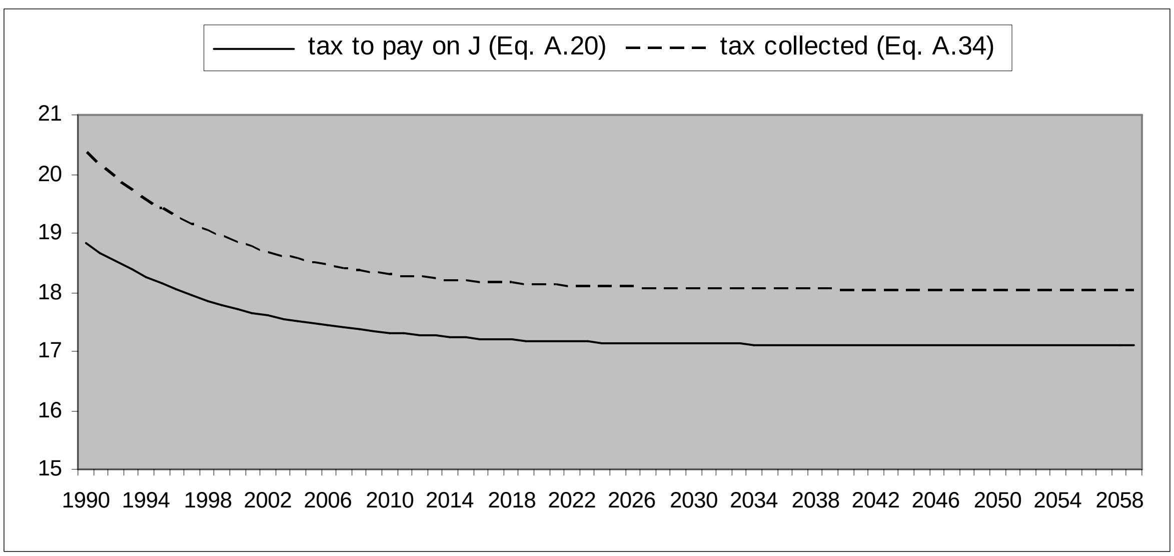
Differences in Endogenous Variables Values for the Export Price Rice Simulation

In DG the first simulation is a simulation where the world export price (pe_t^*) is increased by 10%. The first issue taken up is the leakage that occurs in the DG model. Secondly, some of the graphs of DG are replicated to show how the results differ.

Leakage

To show the leakage, the indirect tax on investment according to DG is shown in Figure 1, first as the amount that need to be paid according to Equation A.20, $tx_t \cdot \left[a_k \cdot PMK_t + (1-a_k) \cdot P_t\right] \cdot J_t$, and then according to the receipts of the government as in Equation A.34, $tx_t \cdot P_t \cdot J_t$. From the figure it is clear that the amount that "needs to be paid" is lower than the amount the "government receives".

Figure 1: Indirect tax receipts for the DG model according to Equation A.20 and Equation A.34 of the Appendix of DG (in bln. Pesos).



For private consumption (in the sequel all variables are per capita variables) it is shown in a graph how the values of the DG model differ with the values of the adjusted model (Figure 2).

Figure 2: Consumption levels for original and adjusted model (dashed line) with an increase in the export price.

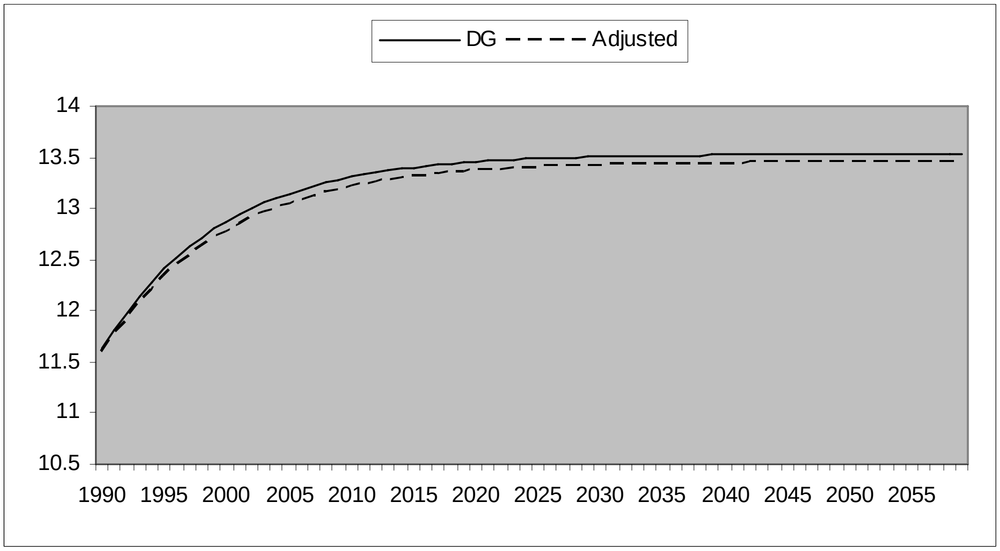


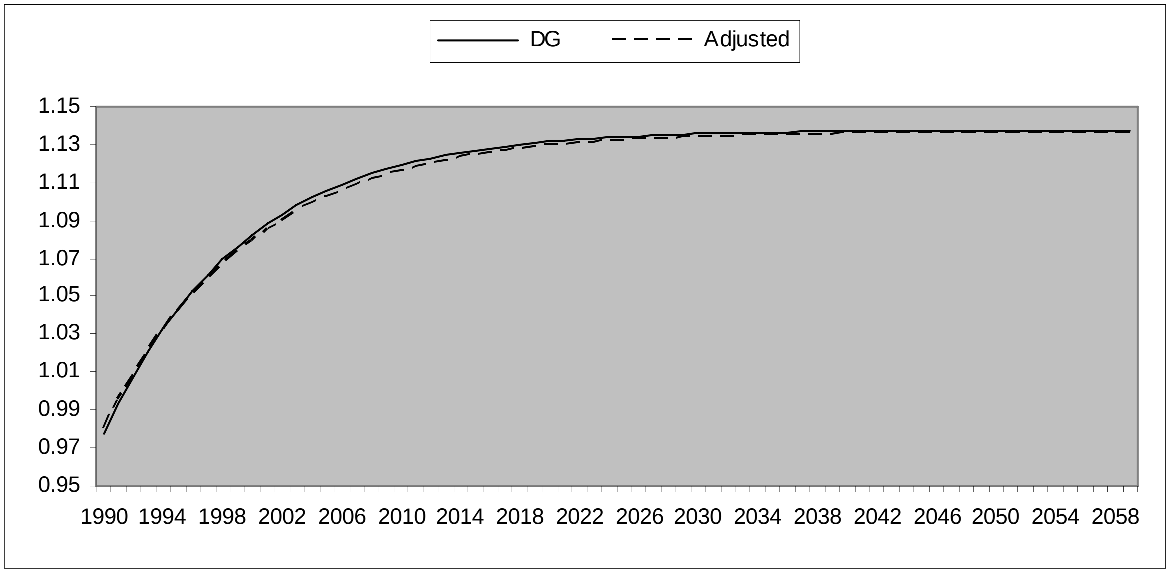
Table 4 shows for consumption and some of the other major endogenous variables the percentage difference between the value with the export price increase and the base run with the adjusted model. The time periods for which the ratio values are shown are chosen in such a way that in the early years more values are shown than in later years. After all, in the early years the possible adjustments take place and in later years the long-term steady state is approached closer and closer. As a result, the values in later years will not adjust that much anymore. The rise in export price leads to larger exports although initially exports go down. The latter allows the country to invest in capital and it is paid for by lending more from the Rest-of-the-World. The adjusted terminal condition with respect to net foreign debt allows for this to happen.

Table 4: Policy outcome for adjusted model of export price increase (percentage increase with respect to base run of adjusted model).

	1990	1991	1992	1994	1998	2004	2014	2031	2059
C_t	-1.968	-0.495	0.845	3.170	6.659	9.869	12.312	13.436	13.658
J_t	24.656	24.368	24.080	23.528	22.574	21.565	20.715	20.301	20.188
E_t	-6.079	-4.662	-3.372	-1.134	2.227	5.321	7.676	8.761	8.970
MK_t	24.656	24.368	24.080	23.528	22.574	21.565	20.715	20.301	20.188
MN_t	0.000	1.142	2.178	3.971	6.650	9.102	10.961	11.815	11.978
$DEBT_t$	0.000	2.353	4.505	8.262	13.953	19.246	23.311	25.200	25.586
K_t	0.000	1.856	3.553	6.517	11.012	15.197	18.413	19.902	20.188
TAX_t	20.360	21.018	21.611	22.626	24.120	25.462	26.465	26.921	27.005

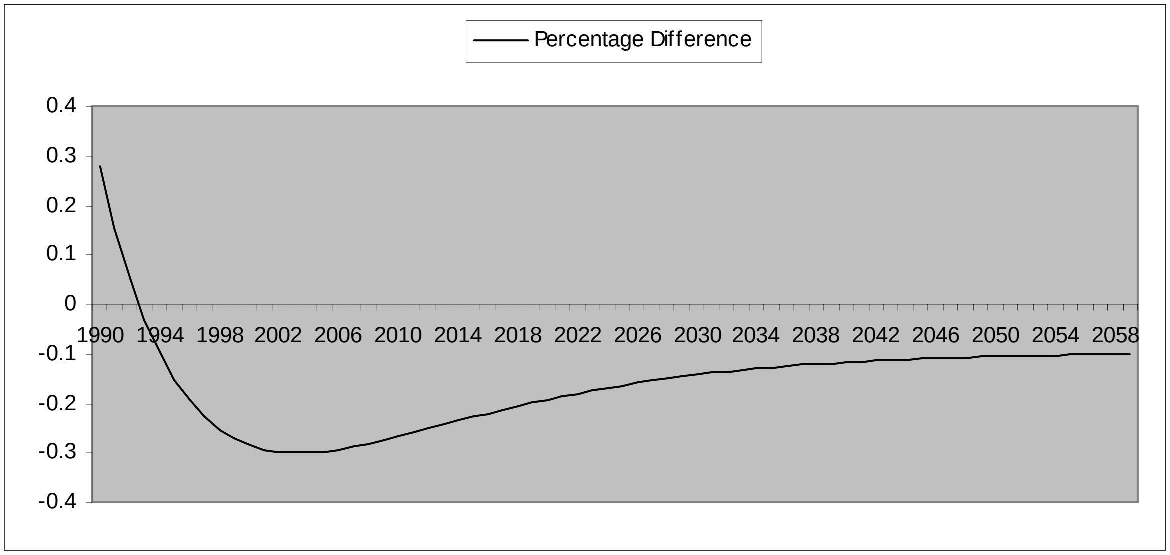
The ratio value for private consumption with respect to the base run solution does not differ that much for the adjusted model and the original model (Figure 3). The ratio is defined as the value of private consumption with the higher export price divided by the base run value for private consumption. This is the replication of Figure 1 in DG on page 694.

Figure 3: Ratio of consumption with respect to base run consumption for DG models and adjusted model (dashed line).



As the two curves lie very close to each other the percentage difference between the two curves is presented:

Figure 4: Percentage difference between the ratio values for the adjusted and the original model in the case of private consumption.



Initially the ratio for the adjusted model has a higher value than the ratio for the original DG model. For later years it is the other way around and the adjusted model has a lower ratio value. Due to the proposed alterations of the model of DG a (slightly) different outcome for private consumption is found. This replicates Figure 2 in DG on page 695 as shown in Figure 5.

Figure 5: Ratio of investment with respect to base run investment for DG and adjusted model (dashed line).

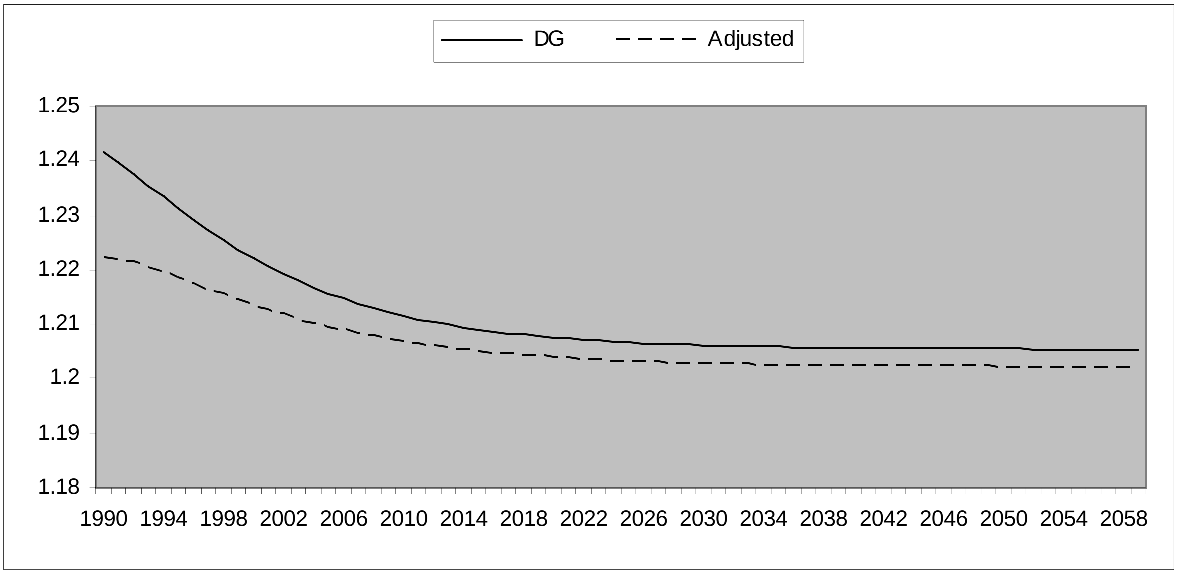


Figure 3 on page 696 is with the ratio value for the real shadow price of capital (q_t / PK_t) over time and this ratio looks like the graphs shown in Figure 6.

Figure 6: Ratio of q_t over PK_t with respect to base run q_t over PK_t for DG and adjusted model (dashed line).

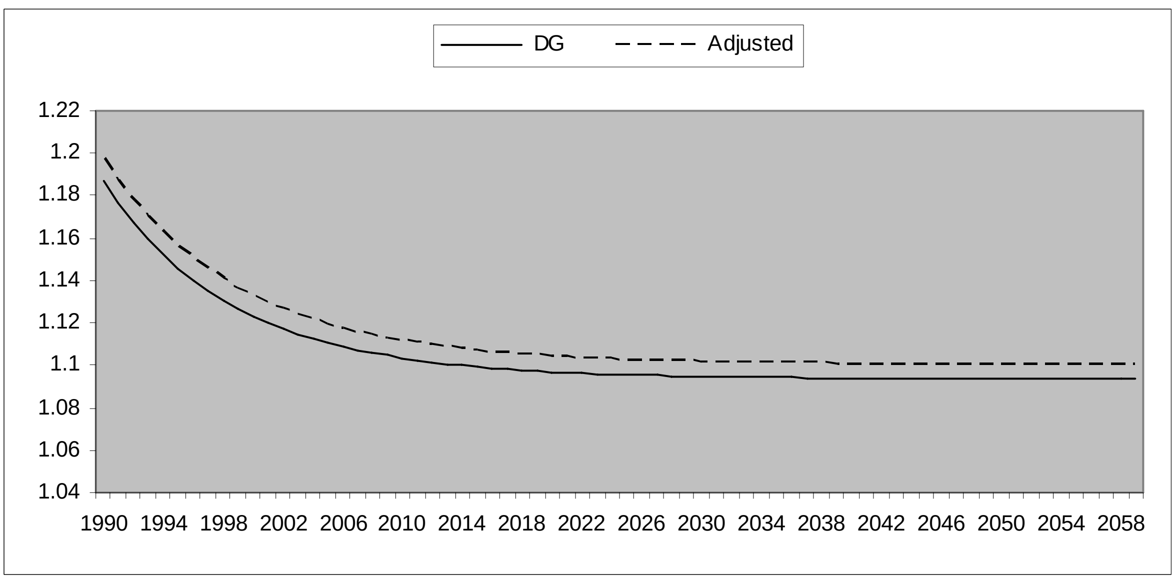
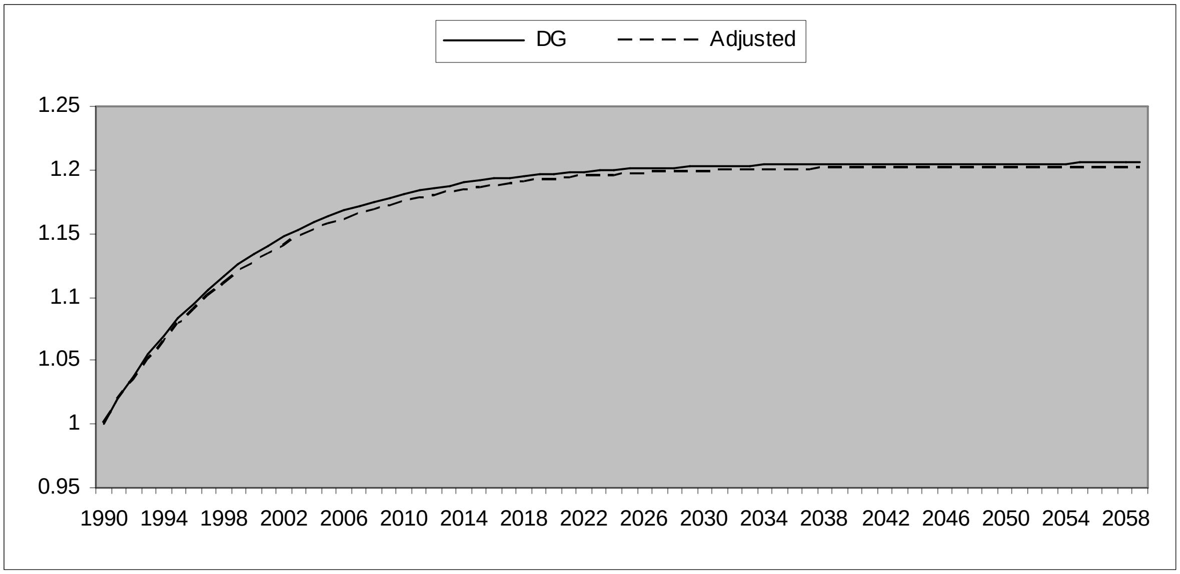


Figure 7 shows how the ratio differs between the DG and adjusted model for the stock of physical capital to replicate Figure 4 of DG on page 697.

Figure 7: Ratio of capital with respect to base run capital for DG and adjusted model (dashed line).



To complete the comparisons of the graphs of DG with the graphs for the adjusted model, Figure 8 shows how the ratio changes for the level of exports (see Figure 6 of DG on page 699).

Figure 8: Ratio of exports with respect to base run exports for DG and adjusted model (dashed line).

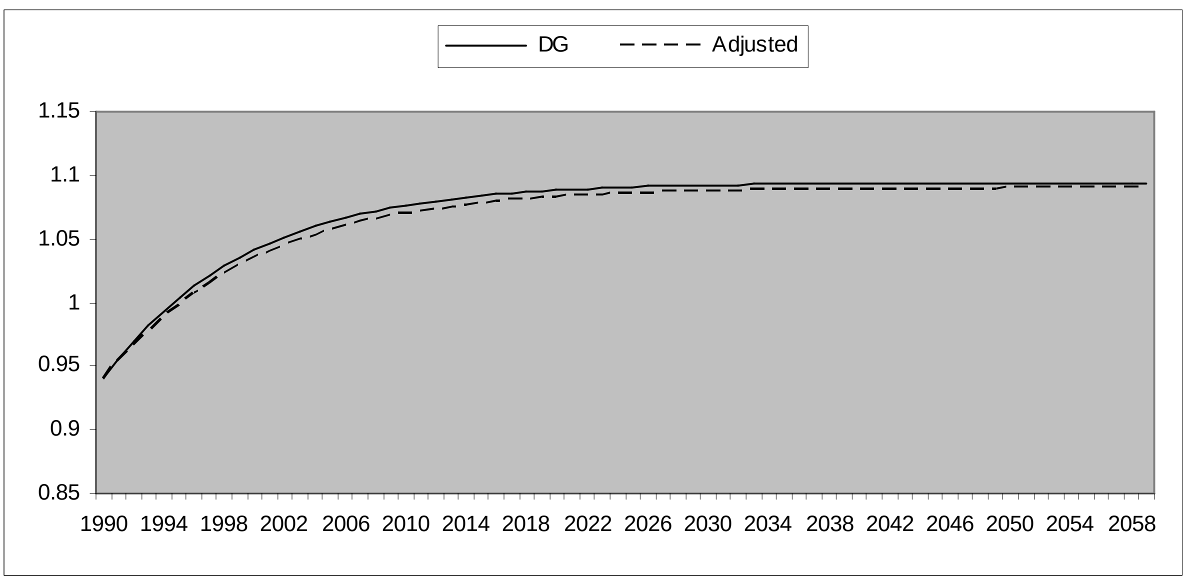
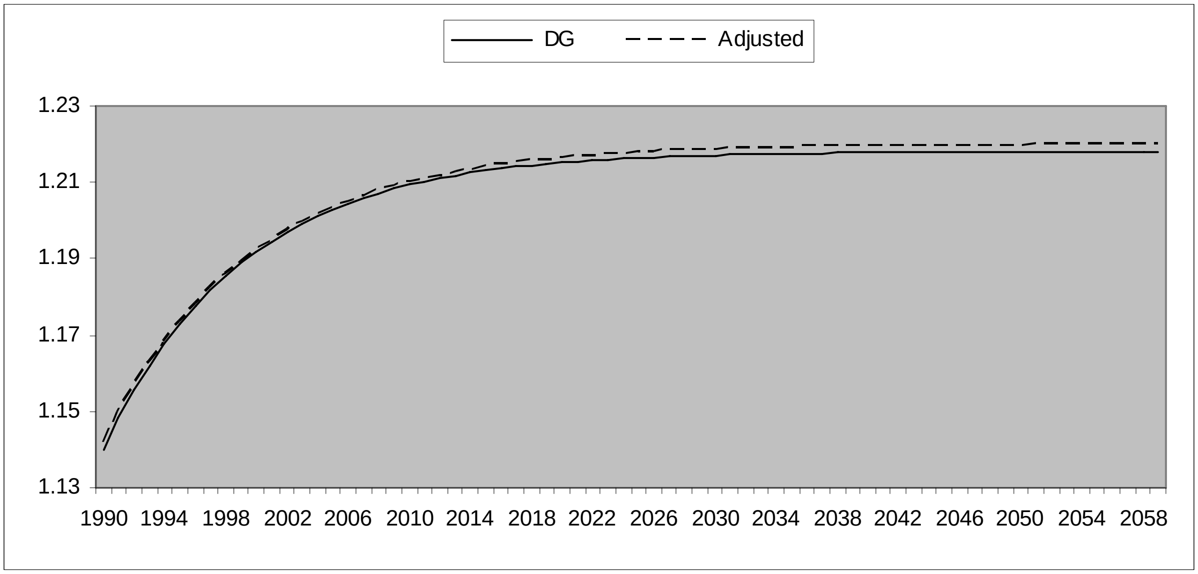


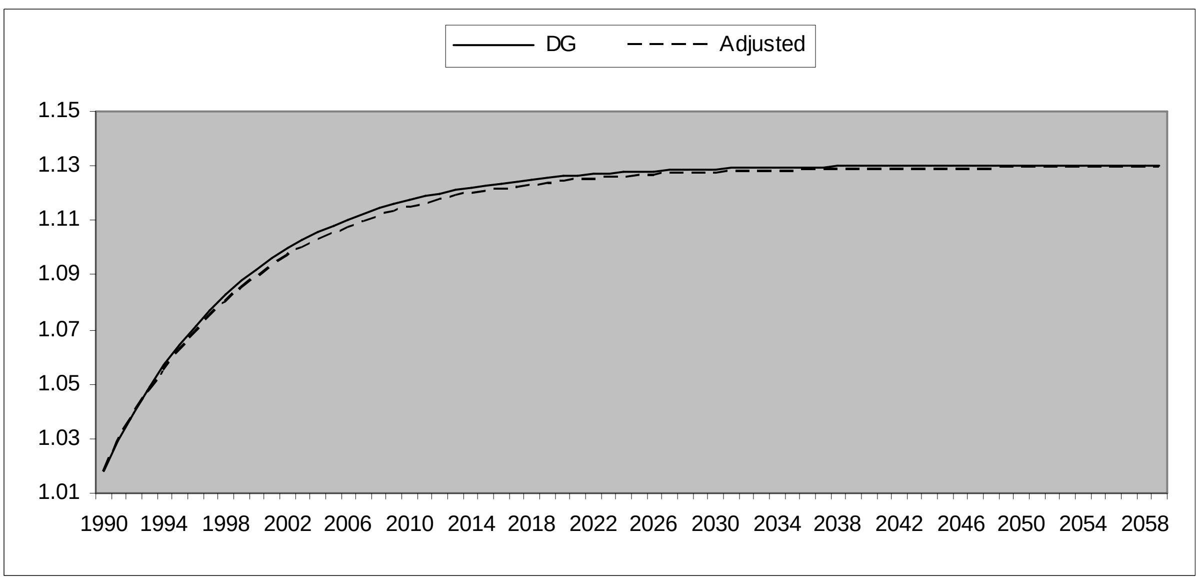
Figure 9 with the level of imports of final goods (see Figure 6 of DG on page 699).

Figure 9: Ratio of final good imports with respect to base run imports for DG and adjusted model a(dashed line).



And finally, in Figure 10 the level of domestic production (see Figure 6 of DG on page 699).

Figure 10: Ratio of domestic production with respect to base run domestic production for DG and adjusted model (dashed line).



5. Concluding remarks

Close inspection of the model equations of Devarajan and Go (1998) reveal that there is a possible (and likely) leakage in that the imported capital good is taxed twice, yet the tax receipts are not fully accounted for in the budget constraint for the government. Additionally, one of the proposed terminal conditions is implied by the other equations of the model due to Walras' Law and, therefore, a terminal condition is missing. By rewriting the equation of the price for the capital good, the imported capital good is not taxed twice. Consequently, the equation describing the tax receipts of the government also needs a small amendment. A terminal condition is added to the model with respect to one of the stock variables in the model (net foreign debt) that assures that the long-term characteristics of the model are well behaved. The adjusted model is recalibrated with the same data set of the Philippines for the year 1990 as in DG. The differences in parameters, exogenous variables and endogenous variables are presented and for the latter some graphs of DG are replicated. It is shown that in the original DG model leakage occurs and the simulation results of the adjusted model are slightly different to the results of the DG model.

Although the differences in simulation results turn out to be minor, the leakage poses a bigger problem. Leakage leads to the fact that the numbers do not add up correctly. Another problem is that if the DG model with leakage is extended, the resulting model will have the same leakage, leading to discrepancies, although the sizes of these discrepancies are not known beforehand. The missing terminal condition also completes the model in that the set of model equations is now square and the number of unknown variables is the same as the number of equations. The model outcomes differ because of the added terminal condition as it allows a country to borrow more or build up foreign assets. With these changes the model is now restored to its intended integrity and provides a very useful framework for policy analysis. The latter is especially useful whenever future research is aimed at extending the model.

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Appendix: The Equations of the Adjusted Model

This appendix contains the total set of equations of DG together with the proposed changes. Equation 4 can be left out as it is implied by Walras' Law (see Section 3). The typing errors in the Appendix of DG have also been corrected. The first pertains to Equation A.16 of DG which has an expression for the export price and from an accounting point of view this equation should read like Equation 23. Although, if the export tax rate te_t is close to zero the two equations are approximately the same. The second typing error is with the exponent in Equation A.31 of DG and this is corrected in Equation 40. Equation A.6 of DG should have a plus sign in front of the term that starts with PK_t, see Equation 11. Finally, the government transfers in Equation A.35 should appear with a minus sign, see Equation 14.

Proposed Set of Model Equations

$$\frac{C_{t+1}}{C_t} = \left(\frac{PC_{t+1}}{PC_t} \cdot \frac{1+\rho}{1+r_{t+1}^c} \right)^{-\frac{1}{\nu}} \tag{1}$$

$$YH_t = w_t \cdot L_t + r_k(t) \cdot K_t + GTRS_t \cdot P_t + FTRS_t \cdot er - i^* \cdot DEBT_t \cdot er \tag{2}$$

$$Y_t = (1 - ty_t) \cdot \left[YH_t - \left(PK_t \cdot J_t - B_t \cdot er - SAV_t^{Gov} \right) \right] \tag{3}$$

$$Y_t = PC_t \cdot C_t \tag{4}$$

$$J_t = I_t \cdot \left[1 + \theta \left(\frac{I_t}{K_t} \right) \right] \tag{5}$$

$$\theta(x_t) = \left(\frac{\beta}{2} \right) \cdot \frac{(x_t - \alpha)^2}{x_t} \tag{6}$$

$$x_t = \frac{I_t}{K_t} \tag{7}$$

$$\frac{I_t}{K_t} = \alpha + \frac{1}{\beta} \cdot Q_t^T \tag{8}$$

$$Q_t^T = \frac{q_t}{PK_t} - 1 \tag{9}$$

$$r_t^p \cdot q_t = R_k(t) + (q_{t+1} - q_t) - \delta \cdot q_{t+1} \tag{10}$$

$$R_k(t) = r_k(t) + PK_t \cdot \left(\frac{I_t}{K_t} \right)^2 \cdot \theta' \left(\frac{I_t}{K_t} \right) \tag{11}$$

$$K_{t+1} = (1 - \delta) \cdot K_t + I_t \tag{12}$$

$$
\begin{aligned}
TAX_t = {} & tmc_t \cdot (MC_t \cdot pmc_t^* \cdot er) + \\
& tmk_t \cdot (MK_t \cdot pmk_t^* \cdot er) + \\
& tmn_t \cdot (MN_t \cdot pmn_t^* \cdot er) + \\
& te_t \cdot \left(E_t \cdot pe_t^* \cdot er\right) + \\
& tx_t \cdot \left[P_t \cdot \left(C_t + G_t + J_t \cdot (1 - a_k)\right)\right] + \\
& ty_t \cdot \left[YH_t - (PK_t \cdot J_t - B_t \cdot er - SAV_t^{Gov})\right]
\end{aligned}
\tag{13}
$$

$$TAX_t + (ts_t/(1 + ts_t)) \cdot PQ_t \cdot Q_t = G_t \cdot PC_t + GTRS_t \cdot P_t + SAV_t^{Gov} \tag{14}$$

$$pmc_t^* \cdot MC_t + pmk_t^* \cdot MK_t + pmn_t^* \cdot MN_t = pe_t^* E_t - i^* \cdot DEBT_t + FTRS_t + B_t \tag{15}$$

$$DEBT_{t+1} = DEBT_t \cdot \left(1 - d_{adj}\right) + B_t \tag{16}$$

$$L_t = \overline{LS_t} \tag{17}$$

$$X_t = C_t + G_t + J_t \cdot (1 - a_k) \tag{18}$$

$$r_t^p = i^* + \frac{e_{t+1}^p - e_t^p}{e_t^p} \tag{19}$$

$$e_t^p = \frac{PE_t}{PD_t} \tag{20}$$

$$r_t^c = i^* + \frac{e_{t+1}^c - e_t^c}{e_t^c} \tag{21}$$

$$e_t^c = \frac{PMC_t}{PD_t} \tag{22}$$

$$PE_t = pe_t^* \cdot \left(1 - te_t\right) \cdot er \tag{23}$$

$$PMC_t = pmc_t^* \cdot \left(1 + tmc_t\right) \cdot er \tag{24}$$

$$PMK_t = pmk_t^* \cdot \left(1 + tmk_t\right) \cdot er \tag{25}$$

$$PMN_t = pmn_t^* \cdot \left(1 + tmn_t\right) \cdot er \tag{26}$$

$$PC_t = P_t \cdot \left(1 + tx_t\right) \tag{27}$$

$$PK_t = a_k \cdot PMK_t + (1 - a_k) \cdot P_t \cdot (1 + tx_t) \tag{28}$$

$$MK_t = a_k \cdot J_t \tag{29}$$

$$MN_t = a_n \cdot Q_t \tag{30}$$

$$PVA_t = PQ_t / (1 + ts_t) - a_n \cdot PMN_t \tag{31}$$

$$X_t = \alpha_c \cdot \left[\delta_c \cdot MC_t^{-\rho_c} + \left(1 - \delta_c\right) \cdot D_t^{-\rho_c} \right]^{-1/\rho_c} \tag{32}$$

$$P_t \cdot X_t = PD_t \cdot D_t + PMC_t \cdot MC_t \tag{33}$$

$$\frac{MC_t}{D_t} = \left[\frac{\delta_c}{1 - \delta_c} \cdot \frac{PD_t}{PMC_t} \right]^{1/(1+\rho_c)} \tag{34}$$

$$Q_t = \alpha_e \cdot \left[\delta_e \cdot E_t^{\rho_e} + \left(1 - \delta_e\right) \cdot D_t^{\rho_e} \right]^{1/\rho_e} \tag{35}$$

$$PQ_t \cdot Q_t = PD_t \cdot D_t + PE_t \cdot E_t \tag{36}$$

$$\frac{E_t}{D_t} = \left[\frac{1 - \delta_e}{\delta_e} \cdot \frac{PE_t}{PD_t} \right]^{1/(\rho_e - 1)} \tag{37}$$

$$Q_t = \alpha_v \cdot \left[\delta_v \cdot L_t^{-\rho_v} + \left(1 - \delta_v\right) \cdot K_t^{-\rho_v} \right]^{-1/\rho_v} \tag{38}$$

$$PVA_t \cdot Q_t = w_t \cdot L_t + r_k\left(t\right) \cdot K_t \tag{39}$$

$$\frac{L_t}{K_t} = \left[\frac{\delta_v}{1 - \delta_v} \cdot \frac{r_k\left(t\right)}{w_t} \right]^{1/(1+\rho_v)} \tag{40}$$

$$I_{tf} = \delta \cdot K_{tf} \tag{41}$$

$$B_{tf} = d_{adj} \cdot DEBT_{tf} \tag{42}$$

$$r_{tf}^{c} = i^{*} \tag{43}$$

$$r_{tf}^{p} = i^{*} \tag{44}$$

List of parameters:

α_e	Shift parameter in the CES function for X_t
α_t	Shift parameter in the CET function for Q_t
α_v	Shift parameter in the CES function for Q_t
a_n	Coefficient of intermediate imports
a_k	Coefficient of capital imports
δ	Depreciation rate of capital
δ_e	Share parameter in the CES function for X_t
δ_t	Share parameter in the CET function for Q_t
δ_v	Share parameter in the CES function for Q_t
α	A parameter in the adjustment cost function
β	A parameter in the adjustment cost function
er	Nominal exchange rate (price Numéraire)
i^*	World interest rate
ρ	Rate of consumer time preference
ρ_e	Exponent parameter in the CES function for X_t
ρ_t	Exponent parameter in the CET function for Q_t
ρ_v	Exponent parameter in the CES function for Q_t
pe_t	World export price
pmc_t^*	World price of final imports
pmk_t^*	World price of capital imports
pmn_t^*	World price of intermediate imports
ts_t	Tax rate for domestic firms
te_t	Export tax
ty_t	Direct income tax
tmc_t	Import duty for imported final goods
tmk_t	Import duty for imported capital goods
tmn_t	Import duty for imported intermediate goods
tx_t	Domestic indirect tax rate
d_{adj}	Adjustment term for net foreign debt

List of prices:

P_t	Price of supply
PC_t	Price of domestic supply including indirect tax
PD_t	Price of domestic goods
PE_t	Domestic price of exports

PK_t	Price of capital
PMC_t	Domestic price of final imports
PMK_t	Domestic price of capital imports
PMN_t	Domestic price of intermediate imports
PQ_t	Price of gross output
PVA_t	Price of value added
e_t^p	Real exchange rate for supply
e_t^c	Real exchange rate for demand
q_t	Shadow price of capital
Q_t^T	Tax adjusted Tobin's q
r_t^p	Discount rate for supply
r_t^c	Discount rate for demand
$r_k(t)$	Gross rate of return to capital
w_t	Wage rate

List of endogenous variables:

C_t	Aggregate consumption
J_t	Total investment expenditures, including adjustment cost
D_t	Domestic goods
E_t	Exports
I_t	Investment
K_t	Capital stock
L_t	Labour demand
Y_t	Net income spent on consumption
MC_t	Final imports
MK_t	Capital imports
MN_t	Intermediate imports
Q_t	Value added
$R_k(t)$	Marginal net revenue product of capital
X_t	Aggregate supply
B_t	Foreign borrowings or capital inflows
SAV_t^{Gov}	Government savings
$DEBT_t$	Outstanding net foreign debt

List of exogenous variables:

G_t	Government consumption

$GTRS_t$	Government transfers to households
$FTRS_t$	Foreign remittances
$\overline{LS_t}$	Labour supply at time t

Miscellaneous:

$\Theta(x_t)$	Adjustment cost function
x_t	Investment capital ratio I_t / K_t

Altered Original Equations

The following is a set of equations that are the same as the equations used by DG, but that have alternative formulations as discussed in the main text. It should be noted, that Equation A.47 is actually not listed in DG, but can be found in the GAMS file that accompanies DG text (note that B_0 denotes B_t at time $t = 0$):

$$\begin{aligned} TAX_t = {} & tmc_t \cdot (MC_t \cdot pmc_t^* \cdot er) + \\ & tmk_t \cdot (MK_t \cdot pmk_t^* \cdot er) + \\ & tmn_t \cdot (MN_t \cdot pmn_t^* \cdot er) + \\ & te_t \cdot \left(E_t \cdot pe_t^* \cdot er\right) + \\ & tx_t \cdot \left[P_t \cdot \left(C_t + G_t + J_t\right)\right] + \\ & ty_t \cdot \left[YH_t - (PK_t \cdot J_t - B_t \cdot er - SAV_t^{Gov})\right] \end{aligned} \tag{A.34}$$

$$PE_t = \frac{pe_t^* \cdot er}{1 + te_t} \tag{A.16}$$

$$PK_t = [a_k \cdot PMK_t + (1 - a_k) \cdot P_t] \cdot (1 + tx_t) \tag{A.20}$$

$$Y_{tf} = PC_{tf} \cdot C_{tf} \tag{A.10}$$

$$B_{tf} = B_0 \tag{A.47}$$